Efficient portfolio: market beta and beyond

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Abstract

The rise of alternative beta investment strategies is a recent trend that positions itself on the top of another powerful trend: the growth of passive “beta” investing. Passive investing in market indices began almost 40 years ago and received a major support from the financial theory, as the market capitalization-weighted portfolio was claimed to be the most efficient one by the Sharpe-Lintner Capital Asset Pricing Model (CAPM) back in the 1960s.

Both theory and empirical research have since then accumulated plenty of evidence that questions the validity of the CAPM and efficiency of the market portfolio. With the “one-for-all” market portfolio solution under attack, a family of new ideas on efficient equity investing, the alternative beta strategies, is proposed to investors. This range of investment ideas is very heterogeneous though its common denominator is the attempt to fix the inefficiencies discovered in the market capitalization weighted portfolios.

In this paper we review the reasoning behind the efficiency of the market portfolio, and its flaws. We then discuss the rationales behind the competing alternative beta investment approaches.

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Market Beta Story

"The market portfolio is the only efficient portfolio" teaches us the Capital Asset Pricing Model (CAPM). The CAPM, introduced by Sharpe [1964] and Lintner [1965], has largely shaped the financial investment theory in the last half century. Its conclusions are taught in most, if not all, finance classrooms. Now that a whole generation has lived by it, its main practical conclusion “the market portfolio is the only efficient portfolio” is engraved in our minds. It means that any investor intending to make the best possible investment decision will opt for an investment in the market portfolio, which is a portfolio where the risky assets are weighted by their market capitalizations. According to the CAPM, this is the portfolio that delivers the maximum Sharpe ratio, in other words no other investment offers better reward for a given risk target.

The immediate consequences of the market portfolio’s efficiency are far-reaching. Investors would no longer need to spend time and resources in looking for a tailor made allocation: the best “one-for-all” choice is readily available. The CAPM proves that it is impossible to consistently beat its peers, and it further implies that trying to outperform the rest of the market will generally result in underperforming it. Originally an indicator of general sentiment about the economy, the market capitalization weighted portfolio, has become an investment objective for managers, a benchmark they should at least track closely, the best allocation they should emulate.

Conveniently, buy-and-hold investing in a cap-weighted mix of assets is one of the easiest ways of investing. Once acquired, a share of the market portfolio will remain efficient without the need of active management or periodic portfolio reviews. This led to the birth of the passive investment era. The share of passive investing has grown[^1] to 13.2% since the first passive mutual fund following S&P 500 index was launched by Vanguard in 1975.

But the CAPM conclusions are not limited to the efficiency of the market portfolio. It also provides a pricing model where expected asset returns are proportional to the portion of systematic risk the stocks carry. The CAPM relation between the market sensitivities (betas) and the expected returns is the foundation of the alpha-beta separation in performance analysis. The expected return of any security is given by the following equation:

\[
E[R_i] = R_f + \beta_i (E[R_m] - R_f) \tag{1}
\]

where \(E[R_i]\) is a one-period expected return of an asset \(E[R_m]\) is the market return, \(R_f\) is the return of the risk-free asset, and \(\beta_i\) is the market sensitivity of the asset, i.e. the regression slope of the asset’s excess return on the excess return of the market.

By analogy, when analyzing realized performances, one can determine the alpha, the portion of the performance that cannot be explained by the aggregate market:

\[
\alpha = (R_i - R_f) - \beta_i (R_m - R_f) \tag{2}
\]

The CAPM argues that the expected value of \(\alpha\) is zero if measured over a sufficiently long period.

Reasoning behind the market portfolio efficiency

We will follow the reasoning behind the CAPM. Starting from the hypothesis of market efficiency, we will highlight how the model’s assumptions come into play. It will also provide us with a basis on which to discuss the model’s limitations.

[^1]: Based on estimate of index fund and industry global assets in 2010, including ETF. Source: C. Philips, “The Case for Indexing”, Vanguard research paper, 2011.
CAPM assumptions and conclusions

The CAPM’s assumptions

i. investing is a costless and smooth process (no transaction costs, no taxation, possibility to buy fractions of shares);
ii. Investors differentiate investments only by their expected returns and variances. When confronted to a choice, they prefer more profits and less risk;
iii. All investors share the same expectations and these expectations are correct;
iv. Every investor has the ability to borrow or lend any amount at the risk free rate.

If the above assumptions are true, the CAPM implies that:

i. The market portfolio is an efficient portfolio: its allocation provides the only optimal mix of risky assets;
ii. For each asset, its expected return follows a simple linear relationship with the expected return of the market portfolio. This relationship depends solely on the regression slope of the assets returns on the return of the market portfolio: the market beta.

First of all, the CAPM assumes that the markets we operate in are efficient. This is crucial in order to rule out arbitrage trades as eligible investments. Market efficiency (not yet the CAPM-market portfolio efficiency) means that investors act rationally and rely on the available information to form their view about the outcomes of the various investments at their disposal. The information must quickly be spread in the marketplace and investors must be able to act on it. If these two conditions are fulfilled, capital markets will reach an optimal state, an “efficient” state. The efficiency of capital markets is thus a measure of how promptly and rationally they react to new information.

Dissemination of information generally happens through a trading process. Informed traders are in a better position to assess the true value of a stock than uninformed traders. If they have the liberty to act on their views, they will drive the market prices toward the fair values and information will spread in the marketplace as the price moves. When markets are efficient, there is no arbitrage trade left and the market as a whole is informed. If a asymmetry in information does arise, it would only exist for a short span of time when arbitrageurs would work to bring the market prices back the equilibrium. The resulting prices are “fair” in the sense that two informed parties would willingly enter in an exchange trade at this price. Thus by assuming markets are efficient, we also assume that market prices are correct at any time.

Once the agents are sufficiently informed, it is necessary to formulate their attitude towards investments. Here the CAPM is based on the rational investor profile, defined in the work of Markowitz on optimal portfolio selection that appeared few years before the work of Sharpe and actually inspired him to construct a pricing model for a rational investor. In this setup investors think of an investment only in terms...
of its final payoff. For instance, an investor will not consider the reluctance he can have in the activities of a certain company, but will focus only on the financial aspects of the trade. The preferences of such an investor can be described through a utility function, which assigns scores to the payoff of an investment. A rational agent will seek to maximize the utility derived through an investment. Because of the uncertainty surrounding the actual terminal values of the investments, one has to think in terms of expected utility and integrate in the problem the probability distribution of terminal values of the considered investment. To arrive to the Markowitz "mean-variance" framework, one needs to assume that either the utility function is quadratic, or the return distribution is normal. Then only two parameters of the payoff's distribution matter: its mean – the expected return on the investment – and its variance – a measure of the expected risk.

Finally, our rational investor acts responsibly when allocating the capital at his disposal. It means assigning more value to the more profitable investment when choosing between two equally risky opportunities, and more value to the less risky investment when choosing between two equally profitable opportunities. We call this behavior responsible because this investor will only choose to bear more risk if compensated for it. In the Markowitz mean-variance scheme, a set of efficient risky portfolios can be built, each efficient portfolio representing an allocation with a maximal return for a given level of risk. Then, an investor with a given risk tolerance simply has to pick up from this efficient frontier a portfolio that is suitable in terms of risk.

Further, the CAPM showed that in the presence of a risk-free asset the efficient frontier of risky portfolios becomes redundant. There is a unique portfolio on the efficient frontier that is superior to all the other portfolios. It is called the tangent portfolio, as it is located on the tangent line connecting the risk-free asset and the efficient frontier. This portfolio provides the highest Sharpe ratio, and thus all rational investors should hold a mix of this portfolio and the risk-free assets if they want to maximize their risk-to-reward ratio.

Example 1: Risky investments We first study the case of an investor that must allocate his capital between two risky investments. Based on expectations about the return and variance of these two opportunities, we can compute expected means and variances for any combination of the investments. Using expectations from the table [1], we can draw the expectations of all portfolios based on different allocations between these two assets.

Among the different allocations on the Figure one is of particular importance: the one which has the lowest expected variance, portfolio C. This portfolio divides possible allocation in two sets: the segment AC, "inefficient allocation" and the segment CB "efficient allocation". A portfolio belongs to the efficient set, if no other portfolio shows higher return for the same amount of risk. A rational investor would never choose a portfolio outside of this efficient set. The portfolio C would be chosen by an investor focusing solely on risk minimization. The portfolio B would be chosen by an investor focusing more on the expected return and ready to assume the corresponding risk. All other portfolios on the segment CB correspond to optimal portfolios for investors with intermediate levels of risk aversion.

Example 2: Risk-free and risky assets We introduce cash as a third asset. This brings leverage as a new parameter in the choice of the optimal portfolio. In order to derive the CAPM conclusions, we will assume an investor has full control on its leverage, being able to borrow and lend unlimited amounts of cash at
Table 1: Expectations about the assets A and B

<table>
<thead>
<tr>
<th>Asset</th>
<th>Expected Return (Risk free rate 3%)</th>
<th>Expected Volatility</th>
<th>Expected Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5%</td>
<td>12%</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>7%</td>
<td>15%</td>
<td>50%</td>
</tr>
</tbody>
</table>

Figure 1: Case of two risky assets without borrowing or lending. On top, the representation of the efficient portfolios in the "mean-variance" space, on the bottom, the representation of efficient portfolios in the "allocation space".

The presence of a risk-free asset modifies the opportunity set available to the investor. As illustrated on the Figure 2, given a portfolio P, all combinations of P and the risk free asset R (leveraged versions of P) will fall on the line passing through R and P in the mean variance plane. This line is often called a "Capital Allocation Line" (CAL). Portfolios on the same CAL will share the same Sharpe ratio. To get an investment with a certain level of risk, one now has at disposal several opportunities: invest in a risky portfolio with the given level of risk, invest in a leveraged version of a less risky portfolio, or invest in a de-leveraged version of a more risky portfolio.

As one sees on the Figure 3, among all the CALs there is one line that lays above all the others: the one that is a tangent line to the frontier of risky portfolios. It crosses the fron-
Figure 2: Illustration of a Capital Allocation Line in the "mean-variance" space

Figure 3: Case of two risky assets with unlimited borrowing and lending. On the left, the representation of efficient portfolios in the "mean-variance" space, on the right, the representation of efficient portfolios in the "allocation space"
tier only in one point, the portfolio T, called the tangent portfolio. The tangent portfolio thus has the highest Sharpe ratio among all the risky portfolios, and a combination of the portfolio T with the risk-free rate, the Capital Market Line, is the new efficient investment set. All portfolios on the new efficient frontier RT are leveraged versions of the portfolio T.

Now the conclusions of the CAPM unfold quickly if we assume uniformity in the borrowing power and expectations of all investors. If all investors share the same expectations they will all deduct the same allocation for the tangent portfolio T and the associated Capital Market Line. If all investors can freely borrow or lend money, they will further agree that all efficient portfolios are leveraged versions of T. All investors would thus invest in a combination of the portfolio T and the risk free asset. The market portfolio, M, - the aggregate of all investors’ portfolios – will itself be identical to the portfolio T. The market portfolio is thus efficient.

Starting from the hypothesis of efficient markets, we have had to make a series of assumptions in order to reach the conclusion of the CAPM. It is these assumptions that we have to keep in mind when using the powerful tools provided by CAPM. We cannot take shortcuts because once an assumption is violated, the final result, namely the efficiency of market portfolio, does not hold any more.

### Market Beta Critics

From its debut, the CAPM and efficient market portfolio standpoint have firmly entered the investment community’s mind. Spectacular growth of passive investments, performance analysis based on alpha-beta separation, building financial forecasts from the market betas, these are only some obvious signs of the triumph of the CAPM and its applications. At the same time, academics and market practitioners undertook numerous empirical studies testing the predictions of the CAPM and the efficient market hypothesis. These studies challenged many of the CAPM conclusions.

From a theoretical standpoint the attack on the CAPM came first of all through the unrealistic nature of some of its assumptions. The real-world markets can be hardly approximated by a homogeneous group of investors with similar views and no investor-specific constraints, leaving apart the investors rationality questioned by the behavioral finance. Another weak CAPM point turned out to be the replicability of the "true" market portfolio. Indeed, CAPM advocates efficiency of a global market portfolio, aggregating all the possible risky holdings investors can have. In reality investors can access only "market proxies", often in the form of regional equity indices. In 1975 Richard Roll even stated that the true market portfolio is unobservable. Finally, an important pricing anomaly was discovered in the 1990s, following the work of Fama and French [1992]. It is widely admitted nowadays that risk factors other than the market impact stock returns, such as value, size, momentum, and later on, volatility.

### What if CAPM assumptions fail

Among the assumptions one has to make in order to derive the conclusions of the CAPM, is a conjecture that investors could borrow unlimited amounts of cash at a risk free rate. This was crucial in the demonstration of market portfolio efficiency as it allowed us to find the one and only one risky portfolio that all investors could agree upon. It can be shown, see for example Markowitz [2005], that if this assumption does not hold the market portfolio is no longer neither optimal nor efficient. We use the same example of two risky assets. This time we will limit the leverage between 0 and
2. We represent all available investment opportunities in the "mean variance" space (Figure 4, left panel), and determine the efficient set. The efficient frontier is no longer reduced to the Capital Market Line and the tangent portfolio T. Because leverage is limited, investors that seek returns above those provided by the twice leveraged version of T, the portfolio TT, will select portfolios that lie on the segment [TT,BB], where BB is a twice leveraged version of the portfolio B. In the allocation space (Figure 4, right panel), it is even clearer that the efficient frontier is divided in two distinct groups: the portfolios on segment [R,TT] and the portfolios on segment [TT,BB]. Respectively, we can distinguish two aggregate portfolios: M1, the average of all portfolios on the segment [R,TT], and M2, the average of all portfolios of the segment [TT,BB]. M1 and M2 also belong to their respective segments, i.e. are themselves efficient. The market portfolio is an aggregate of the portfolios M1 and M2. In the allocation space, it will lay on the segment [M1,M2], out of either efficient subgroup. In this case, the market portfolio is inefficient and no rational investor would want to invest in it.

Empirical tests of CAPM

Starting from the 1970s, researchers made repetitive empirical tests to probe the validity of the CAPM conclusions. A special focus was on testing the formula [1] and its predictions, namely that market beta is related to a positive premium over the risk-free rate, and that the cross-sectional variations in stock returns are explained by their market betas. An excellent review of this research can be found in Fama and French [2004].

As Fama and French indicate, the tests consistently rejected the equivalence between stock expected returns and their CAPM estimations based on market betas. A great amount of variations in stock returns was found to be unrelated to market betas. This important research leads to the discovery of new pricing factors, summarized in the Fama and French 3-factor model (Fama, French [1992]) and later generalizations.

Noisy Market Hypothesis

Another direction of critics was the efficiency of market prices, the so called "Noisy market hypothesis". Rejecting the Efficient Market Hypothesis, i.e. accepting that the market valuations contain deviations from the true underlying values, it can be shown mathematically (see for example Hsu [2006]), that the market portfolio ceases to be the most efficient one. In this case one could build other portfolios with weights that are (to some extent) independent of the market valuations and that will outperform the market portfolio when the market valuations effectively mean-revert to the "true" values. This result motivates researchers and practitioners to search for the alternative weighting schemes that will not contain the errors due to the market "noise".

Roll’s Critique

The famous Roll’s critique (1975) stated that the true market portfolio is unobservable, because in order to assemble it one should include all existing risky assets, covering all possible securities, but also privately held companies and even human capital. Surprisingly, notwithstanding this critic the optimality of the true market portfolio was "granted" to all portfolios weighted by market capitalization, even small ones consisting for example of regional large-cap stocks.
Empirical Features of the Market Portfolio

Even without the use of complex mathematics, the investment community agrees on some properties of the market capitalization weighted portfolios that are in contradiction with an intuitive definition of an efficient portfolio. Market-capitalization weighted portfolios often are very concentrated. A small fraction of mega-cap companies can represent a dominant part of the portfolio, introducing a strong asymmetry in the weight repartition. There is a tendency in the market-cap weighted portfolios to overweight the stocks that recently outperformed the market, resulting in a trend-following behavior. In its extreme form this leads to the formation of market bubbles, like the dot-com bubble of the late 90s in the US. From 1993 to 1999 the Information Technology sector in the S&P 500 index grew 5-fold, from 5.9% to 29.2%, and then its weight was halved in 2002\(^2\) to 14.31%. One more feature is the tendency of market capitalization weighted portfolios to overweight growth stocks, that makes the portfolio tilted towards growth factor and away from the value factor. Altogether, the arguments and evidence of non-efficiency of the "market" portfolio are rather convincing. Still, building "better" portfolios proves to be a very hard task, as is demonstrated by the statistics of active asset managers’ performances with respect to their market-cap weighted benchmarks (see for example Philips [2011]). Indeed, the famous "zero-sum game" argument is often evoked to protect the passive market-following, saying that an average investor in the active funds gets exactly the market return before the fees are paid. Or, if we put it differently, for any winning manager there should be a losing manager on the other side of the trade.

However, there are ways that are currently explored by the researchers and practitioners to systematically exploit the inefficiencies of the market portfolio and the failures of the CAPM. It is worth emphasizing again that in the world "beyond the CAPM" the relations among risk and return characteristics are not necessarily

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\(^2\)Source: Standard & Poor’s
linear, and no universal answer exists for any investor on how to build an efficient portfolio. In the next section we review some of the most widespread "alternative beta" approaches to investing. The words "alternative beta" indicate that the methods in question do not aim at delivering alpha by incorporating unique manager knowledge and skills or superior return forecasts. Rather the alternative beta approaches focus on finding completely systematic rule-based solutions to correct the market-capitalization weighted portfolios, and in this sense are passive (or "beta") investments as well.

Alternative Beta Story

The family of alternative beta, or alternative weighting strategies, is very heterogeneous. By alternative beta we mean the strategies (or portfolios) that are broad and not restricted to some specific investment theme or risk factor (e.g. a bet on an industrial sector, or a value portfolio). The common theme in the alternative beta space is to improve the inefficiencies of market capitalization weighted portfolios and offer more efficient investment solutions, possibly with specific investor believes or constraints in mind. In this sense, the alternative beta portfolios compete with the passive market portfolio just in one aspect: being an efficient investment objective. These are not "market thermometers" as the market-capitalization weighted portfolios are, and are not meant to replace the market portfolio in other areas, as underlying of derivative contracts for example.

We consider here the rationales for some of the alternative weighting approaches, namely equal weighting, minimum variance, risk-parity, economic-scale weighting and diversity weighting. There are more approaches in this family, for more complete review of risk-based alternative beta approaches we invite the reader to see Lee [2010].

Equal Weighting

It is natural to begin with the equal weighted approach for several reasons. First of all, the idea of allocating the same amount to every asset in the portfolio is one of the oldest and by far the simplest one. This strategy was the first to emerge in the passive investing: the first passive investment account was launched in 1973 by Wells Fargo and was actually an implementation of an equal-weight portfolio of NYSE-listed stocks that was later switched to the market-capitalization weights because of operational reasons. The first "alternative beta" ETF was following the S&P 500 Equal Weight index, it was launched by Rydex in 2003. Finally, the equal-weighted portfolio, or "1/N", is a widespread performance analysis benchmark in the academic literature. The equal weight benchmarks are easier to construct, especially if the access to the historical data on market capitalization of stocks is limited. Moreover, in some cases, for example the construction of arbitrage strategies, the market capitalization weights are just not relevant as benchmarks.

The rationale for the equal weighted portfolio is the search for maximal diversification in absence of reliable information on stocks’ future risk and returns. The equal-weight allocation is simple and easy to read, and results in spreading the investment bets evenly across the investment universe. The return of the equal weighted portfolio is also easy to interpret: it is an arithmetic average of all the returns in the portfolio.

Naturally, the equal weighted portfolio corrects the "mega-cap" concentration bias of the capitalization-weighted portfolios and avoids the trend-following behavior, as the portfolio is rebalanced periodically to restore the equal
weights and the past outperformance of a stock relative to the basket does not lead to a superior weight for it. Consequently, the equal weighted portfolio will not follow a market bubble. With 62 Internet companies in the S&P index in late 1999, the total weight of the IT sector in the S&P 500 equal weight was of the order of 62/500 = 12.4 %, that is much lower than the 29.2 % in the S&P 500 portfolio.

The risk profile of the equal weighted portfolio resembles a lot to that of the capitalization-weighted portfolio. Equal weightings brings neither significant increase nor decrease in portfolio long-term volatility with respect to the market-cap counterparts, and equal weighted portfolio generally remains very correlated to the market-capitalization weighted portfolio. To get further insight on possible risks, one should pay attention to the definition of the investment universe that is used to build an equal weighted portfolio. If the universe itself has some significant asymmetries, they will be reflected in the resulting equal weight allocation. For example, a global universe of 5000 stocks will contain a dominant number of small cap stocks that will then dominate the equal weighted portfolio. Or, if there are significant differences in the number of stocks across industries, some industries as a consequence will be underweighted and others overweighted, notwithstanding their weight in the economy.

Possible driver of outperformance of the equal weight strategy with respect to the market cap index is the correction of mega-cap bias. It is often referred as a small-cap bias "driver" of the equal-weighted portfolio performance, but it is true only when the equal weighted portfolio is built on a universe that includes a significant amount of small-cap stocks. Obviously, the large-cap or blue-chip equal-weighted portfolios could not be said to have a small-cap bias.

Our research on European equal weighted portfolios showed that reducing the weight of the large-cap sector and correcting the mega-cap bias inside this sector were the most significant sources of relative outperformance of STOXX Europe 600 Equal Weight index over its market cap counterpart in the period 2003-2010 (see Monnier and Rulik [2011]). One additional performance driver being discussed is the periodic rebalancing that leads to reducing positions in stocks that outperformed and increasing the position of stocks that underperformed, i.e. a contrarian behavior. However, up to our knowledge there is no empirical evidence of the positive performance achieved in this way and in our study we found no significant contribution of this effect to the total excess return of the equal weighted portfolio over the market cap portfolio.

Minimum Variance

As was discussed above, the market-capitalization weighted portfolio in the CAPM framework is thought as an efficient optimized portfolio. This means that it can be constructed via a mean-variance optimization algorithm that takes as an input consensus market forecasts for future returns and the covariance among the stocks. Note that the market-capitalization weighted portfolio is actually an "active" portfolio in the mean-variance sense. It incorporates implicit future return forecasts contained in the stocks' market capitalization and the market weights are optimal only if these forecasts are the best that one can have. There is only one optimal portfolio on the mean-variance frontier that is truly "passive" in its objective and remains on the frontier for any configuration of return forecasts, the Minimum Variance portfolio. Minimum variance is an optimal allocation
that is constructed by minimizing portfolio variance. The minimum variance construction does not use stocks’ expected returns as inputs, and relies only on the covariance matrix. Usually the minimum variance portfolio is depicted as the outmost left point on the mean-variance frontier (see the Figure [1], the less risky and the less performing one. But this picture is very misleading, since it is conditional on some non-homogeneous forecast of future returns. If such a forecast happens to be wrong, the ex-post performance of the minimum variance might be well above the ex-ante optimal portfolios that have higher risk. If ex-post the stocks have similar returns, the minimum variance portfolio will take the place of the efficient tangent portfolio, giving the highest Sharpe ratio.

This makes minimum variance portfolio a pure risk-based solution. The fact that the minimum variance construction does not use return forecasts implies that there is no impact of the return forecasting error on the resulting allocation. As was shown in the academic literature (see for example Chopra and Ziemba [1993]), the negative impact of the return forecasting errors was one of the main causes of disappointing performance of portfolios constructed with the mean-variance optimization. Of course, there are errors that are inevitably contained in the estimation of covariance matrix, and the minimum variance allocation is sensitive to these. Still, the distortions these errors bring to the allocation are much smaller than in the case of forecasted returns, and covariances are by far more stable and predictable in time than returns are.

Contrary to the equal weighted portfolio, the minimum variance approach adds value via portfolio construction. Because of the subtleties of covariance estimation and the choice of constraints that mitigate the covariance estimation risk, the minimum variance methodologies can differ significantly from one provider to another. The explicit use of constrained optimization also offers investors an interesting opportunity to customize their portfolio by including specific constraints, such as maximal weight per stock or per sector, constraints neutralizing exposure to unwanted risk factors, etc… In this sense, the minimum variance framework represents a flexible tool for an investor who does not have specific views on future returns and tries to achieve risk efficiency while maintaining proper risk-management constraints or objectives.

There is a growing body of evidence on performance and characteristics of the minimum variance investing. Over the past three years MSCI, DAX, STOXX and FTSE have created minimum variance versions of their benchmark equity indices. In addition, a growing amount of academic research is dedicated to minimum variance portfolios, where usually some ”test” minimum variance portfolios are studied. All these portfolios share some important properties, as reduced ex-post portfolio volatility with respect to the market-capitalization benchmarks, and low market beta.

If the CAPM were right, the low market beta should have implied smaller return for the minimum variance strategies than that of the market-capitalization benchmark. However, the evidence is showing the opposite situation, the strategies have added significant positive excess return at least over the last 10 years. In part, the empirical outperformance of minimum variance strategies can be explained by the fact that in a multi-period setting the extra volatility reduces the total multi-period return. Think of an ”annualized” geometric average (a multi-period return) versus an arithmetic average (a one-period return), the former is always smaller and the magnitude of the difference is proportional to the half of the return variance. But the multi-period argument still does not tell the whole story. The major part of the outperformance of the minimum variance port-
folios comes from the fact that empirically even in one-period setting the low volatility stocks were found to perform better, or at least not worse, than the high volatility stocks. This means that even arithmetic average of one-month returns is not lower for the low-risk stocks, contrary to what one would expect from the CAPM predictions. The "low-volatility anomaly" in the cross-section of stock returns was reported repeatedly by Haugen, Baker [1991, 2008], who studied the relation between the expected return and the risk measures like total return volatility and idiosyncratic volatility (the volatility of a residual in the factor model of expected stock returns). In a later study Ang et al [2006] has confirmed a negative relation between stock returns in the cross-section and their sensitivity to the market volatility factor (VIX). This means that the stocks that react strongly to the movements in market volatility (and tend to be themselves more volatile) in general underperformed the stocks with lower aggregate risk sensitivity. They have also found a negative relation among stock returns and their idiosyncratic (residual) volatilities that can be only in part explained by their sensitivities to the aggregate market risk.

More research continues in the "low-volatility anomaly" direction, and we did not mean to make a complete overview of the subject here. Even before a consensus of this research emerges, a low-volatility and minimum variance investing represent an interesting opportunity. These are examples of a failure of the CAPM prediction that more risk should be paid off with more return.

**Risk Parity or Equal Risk Contribution**

Risk parity proposes a risk-based portfolio construction method unrelated to the Markowitz-like optimization. The risk parity allocation needs only a covariance matrix as an input, as the minimum variance, but instead of running a full-fledged optimization the method uses an ad-hoc rule and assigns the weights in such a way as to equalize the contributions of all the assets to the portfolio variance. No close-form analytical solution is available to this problem in a general case, but the studies (see for example Maillard, Roncalli and Teiletche [2010]) show that a solution always exists. In the case when all pairwise correlations are equal, the risk-parity weights will be precisely inversely proportional to the stocks’ volatilities (∑i ∼ 1/σi).

The risk parity method leads to a very intuitive allocation: all assets contribute equally to the portfolio risk. It appeals to the investors that are comfortable with the use of risk budgeting rules. The scheme can be also very flexible, as one can redefine the rule for example by requiring parity among risk contributions of industrial sectors rather than individual stocks. The risk of equal risk contribution portfolios is "in between" the market capitalization portfolios and minimum variance portfolios. The stocks with lower volatilities tend to have higher weights in this scheme, but the overall volatility reduction is not as big as for the minimum variance portfolios because in the risk parity portfolio all the stocks are included, even those with very high volatility. Naturally, the risk parity portfolio tends to be well diversified as it includes all the stocks from its investment universe. By overweighting low-risk stocks, the risk parity allocation is well-positioned to profit from the "low-volatility anomaly" as the minimum variance portfolios do.

**Economic Scale Weighting**

Here we turn to a different breed of alternative beta strategies that are not based on risk management considerations or portfolio optimiza-
tion, but rather attempt to add value by more efficient forecast of the stocks valuations than that of the market consensus. This is a convenient way to wrap investor valuations into a systematic investment strategy.

The noise market hypothesis that we already mentioned above, assumes that market valuations come with noise that negatively impacts the ex-post performance of the market capitalization weighted portfolios (see Hsu [2006] and Treynor [2005]). Then, the weighting schemes that are not dependent on market valuations are also free from the market valuation noise, and under certain assumptions this leads to outperformance of market-valuation indifferent allocations. This rationale lies behind the so-called “economic scale” weights of the Fundamental Index introduced in 2005 by Research Affiliates.

Economic scale weighting uses companies’ fundamentals – sales, cash flows, book value and dividends – to construct the stocks valuations, as an alternative to that of the market consensus. As argued in Arnott et al [2010] the errors in such valuations will be independent from the errors present in the stock prices, so when the stocks undervalued or overvalued by the market will have their price corrected the fundamental index portfolio will not suffer, but the market-capitalization weighted portfolio will (see for example Hsu [2006]). This argument is not without flaws, since the outperformance of the Fundamental index depends on a crucial assumption, namely that the errors in the “economic scale” valuations are independent of the market weights. As Kaplan [2008] showed, this is not generally the case and if omitting the error independency assumption the outperformance of the Fundamental index portfolio is no longer guaranteed.

Diversity Weighting

One more approach that can be understood along the lines of noisy market hypothesis is the diversity-based weighting based on the Diversity measure introduced by Fernholz in 1999. The idea of this alternative weighting scheme is to “smooth” extreme bets in the market capitalization weighted portfolio by targeting a greater “diversity”, a measure of portfolio concentration. The higher the portfolio diversity, the more evenly the weights are spread among the stocks in the portfolio. Mathematically, enhancing the diversity amounts to a smoothing power transformation of the market cap weights \( w_i \rightarrow w_i^p, \ 0 < p < 1 \). Such a transformation decreases the gap between the biggest and the smallest weights of the market portfolio and in this way helps to reduce the magnitude of errors in the market valuations. Fernholz et al [2005] argue that if one waits long enough, the diversity-weighted portfolio will outperform the market cap portfolio since the stocks with the biggest market capitalization weights will not be able to continue their ”excess growth” and their weights will be eventually reduced. This weighting scheme is implemented in the Intech Diversity index, which is a diversity-weighted version of the S&P 500 index.

The fundamentals-weighted and diversity-weighted portfolios have some very attractive features that the equal weighted and other risk-based portfolios do not have. Being based on economic size or directly on market weights, these portfolios enjoy almost the same liquidity and capacity that the market-capitalization weighted portfolio. Finally, all the alternative beta schemes discussed above are purely systematic strategies, needing periodic adjustments as stock prices and fundamentals change.
Alternative beta examples

In the last decade, and even more so recently, index providers and asset managers have striven to propose alternatives to market capitalization weighted indices. For many such indices a backtested history is available, that allows to compare the classical capitalization-weighted indices to some alternative market beta indices over at least the last ten years. As an illustration we consider the performance of three European Indices: the Stoxx® Europe 600 market-capitalization weighted index (MC), the Stoxx® Europe 600 Equal Weight index (EW) and the iStoxx® Europe Minimum Variance index (MV). MC is a free float market capitalization weighted index which will serve as our market portfolio proxy. EW and MV represent two alternative beta strategies highlighted in the previous section – respectively Equal Weighted, and Minimum Variance.

0.1 Risk and performance profile of Equal Weight & Minimum Variance indices

We present the data on a ten year period ranging from 31/12/2001 to 31/12/2011 for the three indices. Figure 5 represents the evolution of the level of the three indices. Starting with the relative performance of the EW versus the MC, we witness a risk adjusted outperformance over the period of study, i.e. a positive alpha. With a strong correlation to the market portfolio and comparable volatility, the beta of the equal weight index is close to one. Other measures of riskiness are also positioned at comparable levels, for instance the maximum drawdown is 58.69% for the MC and 64.01% for the EW. The main driver of the outperformance of the EW is its size-neutral allocation: companies with relatively smaller capitalization receive a significant weight while in the MC portfolio they are dwarfed by the investment in widely capitalized companies. Perhaps counter intuitively, the active component of the strategy – periodically resetting equal weighting – is not a critical element in the performance (as we discussed in the previous section); while this is an essential task to perform for the relevance of the strategy.

The MV index has a radically different risk profile. The track record clearly shows a large reduction of the drawdowns. It exhibits a 40% volatility reduction and a corresponding beta of 0.54 over the period. Though its exposure to market risk is reduced, its performance stays in line with our market portfolio proxy through most of the periods, in contradiction to the CAPM conclusions that would predict it to underperform. The MV benefits from its exposure to low volatility stocks which have consistently beaten their beta-based expectations. If we plot these indices on the mean–variance plane, it is even clearer that the proxy of the market portfolio can be improved upon. Alternatives to market capitalization can provide better risk and return profiles.

According to the formula (1), one can calculate the expected returns for the EW and MV indices, based on their market beta. Using monthly returns over the considered period we find that sensitivity to the market capitalization index (market beta) is of 1.14 for the EW and 0.57 for the MV. With the monthly risk-free rate of 0.1925% and average monthly market returns of 0.1818% over the period, the CAPM would give the expected return of 0.1803% for the EW and 0.1867% for the MV. The realized average returns for the two alternative indices were much higher: 0.41% for the EW and 0.57% for the MV.

Conclusion

As one sees, the alternative beta approaches do not give a unique solution on how to fix the
Table 2: Statistics of alternative beta strategies and the market index benchmark over the period 2001-2010. EONIA used as risk free rate for computations. Source: Bloomberg (Data), Ossiam (Computation)

<table>
<thead>
<tr>
<th></th>
<th>Market index</th>
<th>Equal Weight</th>
<th>Minimum Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annualized Performance</td>
<td>0.73%</td>
<td>2.96%</td>
<td>6.25%</td>
</tr>
<tr>
<td>Volatility (annualized)</td>
<td>21.66%</td>
<td>21.34%</td>
<td>13.02%</td>
</tr>
<tr>
<td>Max Drawdown</td>
<td>-58.69%</td>
<td>-64.01%</td>
<td>-39.71%</td>
</tr>
<tr>
<td>Sharpe Ratio*</td>
<td>-7.21%</td>
<td>3.11%</td>
<td>30.41%</td>
</tr>
<tr>
<td>Correlation vs Benchmark</td>
<td>-</td>
<td>96.77%</td>
<td>89.69%</td>
</tr>
<tr>
<td>Beta</td>
<td>-</td>
<td>95.34%</td>
<td>53.90%</td>
</tr>
<tr>
<td>Annual Alpha</td>
<td>-</td>
<td>2.15%</td>
<td>4.80%</td>
</tr>
</tbody>
</table>

problem with the market portfolio inefficiency. Rather these strategies go back to the investor beliefs and constraints, attempting to build efficient portfolios depending on investors’ objectives and views.

Our review of alternative beta approaches in by no means exhaustive, and we address the reader to Arnott et al [2010] and Lee [2010] for more information.

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Figure 6: Mean variance scatter plot based on 2001-2011 annualized data. Source: Bloomberg (Data), Ossiam (Computation)


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Efficient portfolio: market beta and beyond


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Ossiam is a research-driven French asset management firm (authorized by the Autorité des Marchés Financiers) and specializes in delivering smart beta* solutions. Efficient indexing is at the core of Ossiam’s business model. The firm was founded in response to a post-subprime crisis demand from investors for simplicity, liquidity and transparency. Given the environment, there was a growing need among investors for enhanced beta exposure and risk hedging. Ossiam is focused on the development of innovative investment solutions for investors via a new generation of indices.

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